A COMPARITIVE STUDY FOR FINDING AN OPTIMAL SOLUTION OF ASSIGNMENT PROBLEMS USING HUNGARIAN METHOD AND NEW PROPOSED METHOD

CONTENT

ABSTRACT

In this project, new proposed method is used to obtain solutions of Assignment problem. The procedure of the method is systematically illustrated. To give an extensive account of the method some examples are provided. The results derived by this method will be compared with the results of Hungarian method.

CHAPTER – I

INTRODUCTION

The assignment problem is a special case of the transportation problem in which the objective is to assign a number of resources to the equal number of activities at a minimum cost for maximum profit.

Assignment problem is completely degenerate form of a transportation problem. The units available at each origin (resources) and units demanded at each destination (activity) are all equal to one. That means exactly one occupied cell in each row and each column of the transportation table (i.e.) only n occupied (basic) cells in place of the required n+n-1=(2n-1)

The assignment models are completely specified by its two components, the assignment which represents the underlying combinatorial structure and the objective function to be optimized. It was developed by and published in 1955 by H.Kuhn, who gave the name "Hungarian method" because the method in general based in the earlier works of two Hungarian mathematicians (D.Kőnig and J.Egerváry) and is therefore known as Hungarian method of assignment models.

In this project, "New Proposed Method" is used for solving assignment problem. The corresponding method has been formulated and numerical examples have been considered to illustrate the method. Finally, we compare the optimal solutions among new proposed method and existing Hungarian method.

In chapter II, mathematical form of assignment problem is explored.

In chapter III, algorithm of Hungarian method is described.

In chapter IV, algorithm of new proposed method is elaborated.

In chapter V, some numerical examples have been discussed and compared.

CHAPTER – II

MATHEMATICAL FORM OF ASSIGNMENT PROBLEM

Consider a problem of assignment of n resources (workers) to n activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost (or effectiveness) matrix (Cij) is given as under:

Activity

Available

		A_1	A_2	………………..	A_n	
	R_1	C_{11}	C_{12}	………………..	C_{1n}	1
	R_2	C_{21}	C_{22}	………………..	C_{2n}	1
	.	.	.		.	.
	.	.	.		.	.
Resource	.	.	.		.	.
	R_n	C_{n1}	C_{n2}	………………..	C_{nn}	1
	Required	1	1	………………..	1	

Let P_{ij} denote the assignment of resource (workers) i to activity (jobs) j such that

$$P_{ij} = \begin{cases} 1, \text{If the ith resource is assigned to the jth activity.} \\ 0, \text{If the ith resource is not assigned to the jth activity.} \end{cases}$$

The assignment problem can be mathematically defined as the objective function is to,

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} P_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} Pij = 1 \text{ for all } i$$

$$\sum_{j=1}^{n} Pij = 1 \text{ for all } j$$

where $P_{ij} = 0$, if the i^{th} resource is not assigned to the j^{th} activity.
where $P_{ij} = 1$ if the i^{th} resource is assigned to the j^{th} activity and Cij represents the cost of assignment of worker i to job j.

CHAPTER – III

ALGORITHM OF HUNGARIAN METHOD

In this chapter, an algorithm of Hungarian method is discussed.

Step-1:

Check whether it is balanced assignment problem; if not then introduce a dummy row or column to make it balanced.

Step-2:

Select the smallest number from each row and subtract it from the other numbers in the same row as well as for the columns.

Step-3:

Draw the minimum number of vertical and horizontal lines to cover all 0's in all rows and columns in the matrix.

Let the minimum number of assignments is L and the number of columns or rows is n.

If L = n, the matrix then an optimal assignment can be find, then proceed to step (5).

If L < n then proceed to step (3).

Step-4:

Determine the smallest number in the matrix, not covered by L lines. Subtract this minimum number from all uncovered numbers and add the same number at the intersection of horizontal and vertical lines.

Step-5:

Repeat step (2) and step (3) until L = n.

Step-6:

To find assignment by test assigning all the 0's in the rows and columns. The solution is optimal when assigning one and only one 0 per row and column in given matrix.

Step-7:

Repeat the step (5) until to exactly find one 0 to be assignment in each row (column), then process ends.

Step-8:

Write the numbers that corresponding to the 0's assigned in the previous step in the main matrix and calculate the objective function.

CHAPTER – IV

ALGORITHM OF NEW PROPOSED METHOD

In this chapter, an algorithm of new proposed method is constructed to solve assignment problem.

Step-1:
Find the smallest number (cost) of each row. Subtract this smallest number from every number in that row.

Step-2:
Now add 1 to all element and we get at least one 1's in each row. Then make assignment in terms of 1's. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step-3:
Draw the minimum number of vertical and horizontal lines to cover all 1's in all rows and columns in the matrix.

Let the minimum number of assignments is L and the number of columns or rows is n.

If L = n, the matrix then an optimal assignment can be find, then proceed to step (5).

If L < n then proceed to step (3).

Step-4:
Select the smallest number of the reduced matrix not covered by the lines. Subtract all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new 1's in row and column. Again make assignment in terms of lines.

Step-5:
Repeat step (2) and step (3) until L=n.

Step-6:
To find assignment by test assigning all the 1's in the rows and columns. The solution is optimal when assigning 1 and only one 1 per row and column in given matrix.

Step-7:

Repeat the step (5) until to exactly find one 1 to be assignment in each row (column), then process ends.

Step-8:

Write the numbers that corresponding to the 1's assigned in the previous step in the main matrix and calculate the objective function.

CHAPTER V

NUMERICAL EXAMPLES AND COMPARISION

In this chapter, numerical examples of assignment problems are provided and the result is compared with the Hungarian method.

Example (1): A departmental head has five subordinates, and five tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

	Tasks				
Subordinates	**A**	**B**	**C**	**D**	**E**
I	**10**	**6**	**4**	**8**	**3**
II	**2**	**11**	**7**	**7**	**6**
III	**5**	**10**	**11**	**4**	**8**
IV	**6**	**5**	**3**	**2**	**5**
V	**11**	**7**	**10**	**11**	**7**

Solution:

HUNGARIAN METHOD:

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} 10 & 6 & 4 & 8 & 3 \\ 2 & 11 & 7 & 7 & 6 \\ 5 & 10 & 11 & 4 & 8 \\ 6 & 5 & 3 & 2 & 5 \\ 11 & 7 & 10 & 11 & 7 \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step-3:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

Step-4:

If there is exactly one 0 in a row, mark a square around that 0 entry and draw a vertical line passing through that 0, otherwise skip that row.

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

Step-5:

If there is exactly one 0 in a column, mark a square around that 0 entry and draw a horizontal line passing through that 0 otherwise skip that column.

$$\begin{pmatrix} 7 & 3 & 0 & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & 0 & 3 \\ 4 & \boxed{0} & 2 & 4 & 0 \end{pmatrix}$$

Here, all the 0's are deleted. Hence, it is an optimal solution.

Also, Number of allotments = Number of rows/columns

Hence optimal solution is

Subordinates → tasks

I	→	3
II	→	2
III	→	4
IV	→	3
V	→	7

Total minimum value will be (3+2+4+3+7) i.e., 19 hours.

NEW PROPOSED METHOD:

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} 10 & 6 & 4 & 8 & 3 \\ 2 & 11 & 7 & 7 & 6 \\ 5 & 10 & 11 & 4 & 8 \\ 6 & 5 & 3 & 2 & 5 \\ 11 & 7 & 10 & 11 & 7 \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step-3:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

Step-4:

Adding 1 to all the elements

$$\begin{pmatrix} 8 & 4 & 1 & 6 & 1 \\ 1 & 10 & 5 & 6 & 5 \\ 2 & 7 & 7 & 1 & 5 \\ 5 & 4 & 1 & 1 & 4 \\ 5 & 1 & 3 & 5 & 1 \end{pmatrix}$$

Step-5:

If there is exactly one 1in a row, mark a square around that 1 entry and draw a vertical line passing through that 1 otherwise skip that row.

$$\begin{pmatrix} 8 & 4 & 1 & 6 & 1 \\ \boxed{1} & 10 & 5 & 6 & 5 \\ 2 & 7 & 7 & \boxed{1} & 5 \\ 5 & 4 & \boxed{1} & 1 & 4 \\ 5 & 1 & 3 & 5 & 1 \end{pmatrix}$$

Step-6:

If there is exactly one 1 in a column, mark a square around that 1 entry and draw a horizontal line passing through that 1, otherwise skip that column.

$$\begin{pmatrix} 8 & 4 & 1 & 6 & \boxed{1} \\ \boxed{1} & 10 & 5 & 6 & 5 \\ 2 & 7 & 7 & \boxed{1} & 5 \\ 5 & 4 & \boxed{1} & 1 & 4 \\ 5 & \boxed{1} & 3 & 5 & 1 \end{pmatrix}$$

Now all the 1’s are deleted. Hence, it is an optimal solution.

Number of allotments = Number of rows/columns

Hence, the optimal solution is,

Subordinates	→	Tasks
I	→	3
II	→	2
III	→	4
IV	→	3
V	→	7

Total minimum value will be (3+2+4+3+7) i.e., 19 hours.

Comparison table:

Hungarian method	New proposed method
19	19

Example (2): A Pharmaceutical Company is producing a single product and is selling it through four agencies located in different cities. All of a sudden, there is a demand for the product in another four cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distance is minimized. The distance between the surplus and deficit cities (in kilometers) is given in the following table:

		Deficit cities			
	Men	**A**	**B**	**C**	**D**
Surplus cities	**I**	**8**	**20**	**15**	**17**
	II	**15**	**16**	**12**	**10**
	III	**22**	**19**	**16**	**30**
	IV	**25**	**15**	**12**	**9**

Solution:

HUNGARIAN METHOD:

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} \mathbf{8} & \mathbf{20} & \mathbf{15} & \mathbf{17} \\ \mathbf{15} & \mathbf{16} & \mathbf{12} & \mathbf{10} \\ \mathbf{22} & \mathbf{19} & \mathbf{16} & \mathbf{30} \\ \mathbf{25} & \mathbf{15} & \mathbf{12} & \mathbf{9} \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} \mathbf{0} & \mathbf{12} & \mathbf{7} & \mathbf{9} \\ \mathbf{5} & \mathbf{6} & \mathbf{2} & \mathbf{0} \\ \mathbf{6} & \mathbf{3} & \mathbf{0} & \mathbf{14} \\ \mathbf{16} & \mathbf{6} & \mathbf{3} & \mathbf{0} \end{pmatrix}$$

Step-3:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} 0 & 9 & 7 & 9 \\ 5 & 3 & 2 & 0 \\ 6 & 0 & 0 & 14 \\ 16 & 3 & 3 & 0 \end{pmatrix}$$

Step-4:

If there is exactly one 0 in a row, mark a square around that 0 entry and draw a vertical line passing through that 0, otherwise skip that row.

$$\begin{pmatrix} \boxed{0} & 9 & 7 & 9 \\ 5 & 3 & 2 & \boxed{0} \\ 6 & 0 & 0 & 14 \\ 16 & 3 & 3 & 0 \end{pmatrix}$$

Step-5:

If there is exactly one 0 in a column, mark a square around that 0 entry and draw a horizontal line passing through that 0 otherwise skip that column.

$$\begin{pmatrix} \boxed{0} & 9 & 7 & 9 \\ 5 & 3 & 2 & \boxed{0} \\ 6 & \boxed{0} & 0 & 14 \\ 16 & 3 & 3 & 0 \end{pmatrix}$$

Since the number of allotments is not equal to the number of rows, the solution is not yet an optimal solution.

Step-6:

Now, select the minimum value of the undeleted values. Here, it is 2. Add that value in the intersecting point and Subtract that value in the undeleted cell values

$$\begin{pmatrix} 0 & 7 & 5 & 9 \\ 5 & 1 & 0 & 0 \\ 8 & 0 & 0 & 16 \\ 16 & 1 & 1 & 0 \end{pmatrix}$$

Step-7:

If there is exactly one 0 in a row, mark a square around that 0 entry and draw a vertical line passing through that 0, otherwise skip that row.

$$\begin{pmatrix} \boxed{0} & 7 & 5 & 9 \\ 5 & 1 & 0 & 0 \\ 8 & 0 & 0 & 16 \\ 16 & 1 & 1 & \boxed{0} \end{pmatrix}$$

Step-8:

If there is exactly one 0 in a column, mark a square around that 0 entry and draw a horizontal line passing through that 0 otherwise skip that column.

$$\begin{pmatrix} \boxed{0} & 7 & 5 & 9 \\ 5 & 1 & \boxed{0} & 0 \\ 8 & \boxed{0} & 0 & 16 \\ 16 & 1 & 1 & \boxed{0} \end{pmatrix}$$

Here, all the 0's are deleted. Hence, it is an optimal solution.

Also, Number of allotments = Number of rows/columns

Hence optimal solution is

Surplus cities		Deficit cities
I	→	8
II	→	12
III	→	19
IV	→	9

Total minimum value will be (8+12+19+9) i.e., 48 kilometers.

NEW PROPOSED METHOD:

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} \mathbf{8} & \mathbf{20} & \mathbf{15} & \mathbf{17} \\ \mathbf{15} & \mathbf{16} & \mathbf{12} & \mathbf{10} \\ \mathbf{22} & \mathbf{19} & \mathbf{16} & \mathbf{30} \\ \mathbf{25} & \mathbf{15} & \mathbf{12} & \mathbf{9} \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} \mathbf{0} & \mathbf{12} & \mathbf{7} & \mathbf{9} \\ \mathbf{5} & \mathbf{6} & \mathbf{2} & \mathbf{0} \\ \mathbf{6} & \mathbf{3} & \mathbf{0} & \mathbf{14} \\ \mathbf{16} & \mathbf{6} & \mathbf{3} & \mathbf{0} \end{pmatrix}$$

Step-3:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} \mathbf{0} & \mathbf{9} & \mathbf{7} & \mathbf{9} \\ \mathbf{5} & \mathbf{3} & \mathbf{2} & \mathbf{0} \\ \mathbf{6} & \mathbf{0} & \mathbf{0} & \mathbf{14} \\ \mathbf{16} & \mathbf{3} & \mathbf{3} & \mathbf{0} \end{pmatrix}$$

Step-4:

Adding 1 to all the elements

$$\begin{pmatrix} \mathbf{1} & \mathbf{10} & \mathbf{8} & \mathbf{10} \\ \mathbf{6} & \mathbf{4} & \mathbf{3} & \mathbf{1} \\ \mathbf{7} & \mathbf{1} & \mathbf{1} & \mathbf{15} \\ \mathbf{17} & \mathbf{4} & \mathbf{4} & \mathbf{1} \end{pmatrix}$$

Step-5:

If there is exactly one 1 in a row, mark a square around that 1 entry and draw a vertical line passing through that 1 otherwise skip that row.

$$\begin{pmatrix} \boxed{1} & 10 & 8 & 10 \\ 6 & 4 & 3 & \boxed{1} \\ 7 & 1 & 1 & 15 \\ 17 & 4 & 4 & 1 \end{pmatrix}$$

Step-6:

If there is exactly one 1 in a column, mark a square around that 1 entry and draw a horizontal line passing through that 1, otherwise skip that column.

$$\begin{pmatrix} \boxed{1} & 10 & 8 & 10 \\ 6 & 4 & 3 & \boxed{1} \\ 7 & \boxed{1} & 1 & 15 \\ 17 & 4 & 4 & 1 \end{pmatrix}$$

Number of allotments is not equal to the number of rows. So, it is not yet a optimal solution.

Step-7:

Now, select the minimum value of the undeleted values. Here, it is 3. Add that value in the intersecting point and Subtract that value in the undeleted cell values

$$\begin{pmatrix} 1 & 7 & 5 & 10 \\ 6 & 1 & 0 & 1 \\ 10 & 1 & 1 & 18 \\ 17 & 1 & 1 & 1 \end{pmatrix}$$

Step-8:

Adding one to the elements which are not deleted,

$$\begin{pmatrix} 1 & 8 & 6 & 10 \\ 6 & 2 & 1 & 1 \\ 10 & 1 & 1 & 18 \\ 17 & 2 & 2 & 1 \end{pmatrix}$$

Step-9:

If there is exactly one 1in a row, mark a square around that 1 entry and draw a vertical line passing through that 1 otherwise skip that row.

$$\begin{pmatrix} \boxed{1} & 8 & 6 & 10 \\ 6 & 2 & 1 & 1 \\ 10 & 1 & 1 & 18 \\ 17 & 2 & 2 & \boxed{1} \end{pmatrix}$$

Step-10:

If there is exactly one 1 in a column, mark a square around that 1 entry and draw a horizontal line passing through that 1, otherwise skip that column.

$$\begin{pmatrix} \boxed{1} & 8 & 6 & 10 \\ 6 & 2 & \boxed{1} & 1 \\ 10 & \boxed{1} & 1 & 18 \\ 17 & 2 & 2 & \boxed{1} \end{pmatrix}$$

Now all the 1's are deleted. Hence, it is an optimal solution.

Number of allotments = Number of rows/columns

Hence, the optimal solution is,

Surplus cities Deficit cities

I → 8
II → 12
III → 19
IV → 9

Total minimum value will be (8+12+19+9) i.e., 48 kilometers.

Comparison table:

Hungarian method	New proposed method
48	48

Example (3): Four managers are available to do jobs in five zones. From past records, the time (in hours) that each man takes to do each job is known and given in the following table:

	Zone				
Manager	**A**	**B**	**C**	**D**	**E**
I	**10**	**11**	**4**	**2**	**8**
II	**7**	**11**	**10**	**14**	**12**
III	**5**	**6**	**9**	**12**	**14**
IV	**13**	**15**	**11**	**10**	**7**

Solution:

HUNGARIAN METHOD:

Since the given assignment problem is unbalanced, we have to introduce a dummy column as a fifth row.

Step-1:

The matrix form of the given assignment problem

$$\begin{pmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step-3:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step-4:

If there is exactly one 0 in a row, mark a square around that 0 entry and draw a vertical line passing through that 0, otherwise skip that row.

$$\begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step-5:

If there is exactly one 0 in a column, mark a square around that 0 entry and draw a horizontal line passing through that 0 otherwise skip that column.

$$\begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ 0 & \boxed{0} & 0 & 0 & 0 \end{pmatrix}$$

Since the number of allotments is not equal to the number of rows, the solution is not yet an optimal solution.

Step-6:

Now, select the minimum value of the undeleted values. Here, it is 1. Add that value in the intersecting point and Subtract that value in the undeleted cell values.

$$\begin{pmatrix} 8 & 8 & 1 & 0 & 6 \\ 0 & 3 & 2 & 7 & 5 \\ 0 & 0 & 3 & 7 & 9 \\ 6 & 7 & 3 & 3 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Step-7:

If there is exactly one zero in a row, mark a square around that zero entry and draw a vertical line passing through that zero otherwise skip that row.

$$\begin{pmatrix} 8 & 8 & 1 & \boxed{0} & 6 \\ \boxed{0} & 3 & 2 & 7 & 5 \\ 0 & \boxed{0} & 3 & 7 & 9 \\ 6 & 7 & 3 & 3 & \boxed{0} \\ 1 & 0 & \boxed{0} & 1 & 1 \end{pmatrix}$$

Here, all the 0's are deleted. Hence, it is an optimal solution.

Also, Number of allotments = Number of rows/columns

Hence optimal solution is

Manager → Zone

I → 2
II → 7
III → 6
IV → 7
V → 0

Total minimum value will be (2+7+6+7) i.e., 22 man-hours.

NEW PROPOSED METHOD:

Since the given assignment problem is unbalanced, we have to introduce a dummy column as a fifth row.

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each row and subtract it from each element in that row. The reduced matrix is,

$$\begin{pmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step-3:

Adding 1 to all the elements

$$\begin{pmatrix} 9 & 10 & 3 & 1 & 7 \\ 1 & 5 & 4 & 8 & 6 \\ 1 & 2 & 5 & 8 & 10 \\ 7 & 9 & 5 & 4 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Step-4:

If there is exactly one 1 in a row, mark a square around that 1 entry and draw a vertical line passing through that 1 otherwise skip that row.

$$\begin{pmatrix} 9 & 10 & 3 & \boxed{1} & 7 \\ \boxed{1} & 5 & 4 & 8 & 6 \\ 1 & 2 & 5 & 8 & 10 \\ 7 & 9 & 5 & 4 & \boxed{1} \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Step-5:

If there is exactly one 1 in a column, mark a square around that 1 entry and draw a horizontal line passing through that 1, otherwise skip that column.

$$\begin{pmatrix} 9 & 10 & 3 & \boxed{1} & 7 \\ \boxed{1} & 5 & 4 & 8 & 6 \\ 1 & 2 & 5 & 8 & 10 \\ 7 & 9 & 5 & 4 & \boxed{1} \\ 1 & \boxed{1} & 1 & 1 & 1 \end{pmatrix}$$

Number of allotments is not equal to the number of rows,

So, the solution is not yet optimal,

Step-6:

Now, select the minimum value of the undeleted values. Here, it is 2. Add that value in the intersecting point and Subtract that value in the undeleted cell values.

$$\begin{pmatrix} 9 & 8 & 1 & 1 & 7 \\ 1 & 3 & 2 & 8 & 6 \\ 1 & 0 & 3 & 8 & 10 \\ 7 & 7 & 3 & 4 & 1 \\ 3 & 1 & 1 & 3 & 3 \end{pmatrix}$$

Step-7:

Adding one to the elements which are not deleted values,

$$\begin{pmatrix} 9 & 9 & 2 & 1 & 7 \\ 1 & 4 & 3 & 8 & 6 \\ 1 & 1 & 4 & 8 & 10 \\ 7 & 8 & 4 & 4 & 1 \\ 3 & 1 & 1 & 3 & 3 \end{pmatrix}$$

Step-8:

[If there is exactly one 1 in a row, mark a square around that one entry and draw a vertical line passing through that one otherwise skip that row]

$$\begin{pmatrix} 9 & 9 & 2 & \boxed{1} & 7 \\ \boxed{1} & 4 & 3 & 8 & 6 \\ 1 & \boxed{1} & 4 & 8 & 10 \\ 7 & 8 & 4 & 4 & \boxed{1} \\ 3 & 1 & \boxed{1} & 3 & 3 \end{pmatrix}$$

Now all the 1's are deleted. Hence, it is an optimal solution.

Number of allotments = Number of rows/columns

Hence, the optimal solution is

Manager → Zone

I → 2
II → 7
III → 6
IV → 7
V → 0

Total minimum value will be (2+7+6+7) i.e., 22 man-hours.

Comparison table:

Hungarian method	New proposed method
22	22

Example (4): A manufacturing company has three zones A, B, C and four sales engineers P, Q, R and S for assignment. The engineers are having different sales ability. Since the zones are not equally rich in sales potential, it is estimated that a particular zone will be bring the following sales:

	Zones		
Sales engineer	**A**	**B**	**C**
P	**9**	**26**	**15**
Q	**13**	**27**	**6**
R	**35**	**20**	**15**
S	**18**	**30**	**20**

Solution:

HUNGARIAN METHOD:

Since the given assignment problem is unbalanced, we have to introduce a dummy column as a fourth column.

Step-1:

The matrix form of the given assignment problem is

$$\begin{pmatrix} 9 & 26 & 15 & 0 \\ 13 & 27 & 6 & 0 \\ 35 & 20 & 15 & 0 \\ 18 & 30 & 20 & 0 \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} 0 & 6 & 9 & 0 \\ 4 & 7 & 0 & 0 \\ 26 & 0 & 9 & 0 \\ 9 & 10 & 14 & 0 \end{pmatrix}$$

Step-3:

If there is exactly one 0 in a row, mark a square around that 0 entry and draw a vertical line passing through that 0, otherwise skip that row.

$$\begin{pmatrix} 0 & 6 & 9 & 0 \\ 4 & 7 & 0 & 0 \\ 26 & 0 & 9 & 0 \\ 9 & 10 & 14 & \boxed{0} \end{pmatrix}$$

Step-4:

If there is exactly one 0 in a column, mark a square around that 0 entry and draw a horizontal line passing through that 0 otherwise skip that column.

$$\begin{pmatrix} \boxed{0} & 6 & 9 & 0 \\ 4 & 7 & \boxed{0} & 0 \\ 26 & \boxed{0} & 9 & 0 \\ 9 & 10 & 14 & \boxed{0} \end{pmatrix}$$

Here, all the 0's are deleted. Hence, it is an optimal solution.

Also, Number of allotments = Number of rows/columns

Hence optimal solution is

Sales engineer → zones

I	→	9
II	→	6
III	→	20
IV	→	0

Total minimum value will be (9+6+20) i.e., Rs.35 thousands.

NEW PROPOSED METHOD:

Since the given assignment problem is unbalanced, we have to introduce a dummy column as a fourth column.

Step-1:

The matrix form of the given assignment problem is,

$$\begin{pmatrix} \mathbf{9} & \mathbf{26} & \mathbf{15} & \mathbf{0} \\ \mathbf{13} & \mathbf{27} & \mathbf{6} & \mathbf{0} \\ \mathbf{35} & \mathbf{20} & \mathbf{15} & \mathbf{0} \\ \mathbf{18} & \mathbf{30} & \mathbf{20} & \mathbf{0} \end{pmatrix}$$

Here, the number of rows is equal to the number of columns. Hence, it is a balanced assignment problem.

Step-2:

Find the smallest element in each column and subtract it from each element in that column. The reduced matrix is,

$$\begin{pmatrix} \mathbf{0} & \mathbf{6} & \mathbf{9} & \mathbf{0} \\ \mathbf{4} & \mathbf{7} & \mathbf{0} & \mathbf{0} \\ \mathbf{26} & \mathbf{0} & \mathbf{9} & \mathbf{0} \\ \mathbf{9} & \mathbf{10} & \mathbf{14} & \mathbf{0} \end{pmatrix}$$

Step-3:

Adding 1 to every element

$$\begin{pmatrix} \mathbf{1} & \mathbf{7} & \mathbf{10} & \mathbf{1} \\ \mathbf{5} & \mathbf{8} & \mathbf{1} & \mathbf{1} \\ \mathbf{27} & \mathbf{1} & \mathbf{10} & \mathbf{1} \\ \mathbf{10} & \mathbf{11} & \mathbf{15} & \mathbf{1} \end{pmatrix}$$

Step-4:

If there is exactly one 1 in a row, mark a square around that 1 entry and draw a vertical line passing through that 1 otherwise skip that row.

$$\begin{pmatrix} 1 & 7 & 10 & 1 \\ 5 & 8 & 1 & 1 \\ 27 & 1 & 10 & 1 \\ 10 & 11 & 15 & \boxed{1} \end{pmatrix}$$

Step-5:

If there is exactly one 1 in a column, mark a square around that 1 entry and draw a horizontal line passing through that 1, otherwise skip that column.

$$\begin{pmatrix} \boxed{1} & 7 & 10 & 1 \\ 5 & 8 & \boxed{1} & 1 \\ 27 & \boxed{1} & 10 & 1 \\ 10 & 11 & 15 & \boxed{1} \end{pmatrix}$$

Now all the 1's are deleted. Hence, it is an optimal solution.

Number of allotments = Number of rows/columns

Hence, the optimal solution is,

Sales engineer → zones

I → 9
II → 6
III → 20
IV → 0

Total minimum value will be (9+6+20) i.e., Rs.35 thousands.

Comparison table:

Hungarian method	New proposed method
35	35

CONCLUSION

In this project, New proposed Method is applied to solve assignment problems.

It is observed that,

- All kinds of assignment problems can be addressed using this method
- New proposed method is easier than the known Hungarian method
- Numerical examples and comparison table show that the final output of the new proposed method is similar to an solution to produced when using the Hungarian method

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